Cutoff Wavenumbers in Truncated Waveguides

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***Abstract*—Truncated waveguides are used in some microwave components. The cutoff wavenumbers in truncated circular waveguide and truncated square waveguide are desired but usually difficult to be computed. In this paper, they are efficiently determined using the unified method that we proposed earlier. Both TE-polarized and TM-polarized modes are studied. To demonstrate the applicability and correctness of this method, two practical examples are considered, one is a truncated circular waveguide and the other is a truncated square waveguide. Results obtained for the two cases are compared with existing data in literature and a good agreement is observed. A further extension of the work is made to compute cutoff wavenumbers of a truncated elliptical waveguide. It is found from the analysis that the cutoff wavenumbers in these irregular waveguides can be computed easily, rapidly, and accurately using the unified method.**

***Index Terms*—Cutoff wavenumbers, truncated waveguides, unified method.**

I. Introduction

TRUNCATED circular and square waveguides have found to have better coupling between the orthogonal dual modes than adding a screw at 45° with respect to electric fields of the two modes. So it has many applications in the design of dual mode cavity filters [1], [2] and waveguide polarizers [3]. Levy presented the formula relating these two kinds of applications and more references may be found in [4]. The methods about analyzing waveguides with regular cross-sections are reviewed in [5], [6]. For truncated circular waveguides, the finite difference method [7], the finite strip method [8], and the perturbation approach [4] have been used to obtain the cutoff wavenumber of the dominant TE mode with an aspect ratio from 0.7 to 1. Truncated square waveguides are also studied in [4]. But there is some discrepancy among above results. In a recent paper, the method of internal matching (MIM) [9] is proposed to extend the aspect ratio from 0 to 1, and for the dominant mode and the higher-order TE-polarized and TM-polarized modes in truncated circular waveguides.

In our previous analysis, a unified method has been proposed to analyze a large class of hollow conducting waveguides of different cross sections [10], [11]. This method has been proven to be straightforward, flexible and efficient. Although this method has been widely applied earlier, we do not know if it is applicable to truncated circular/elliptical and square waveguides and what the accuracy is. This motivates the present analysis of truncated elliptical and square waveguides using the unified method in this paper. In the final results, a truncated circular waveguide can be considered as a special case of the truncated elliptical waveguides.

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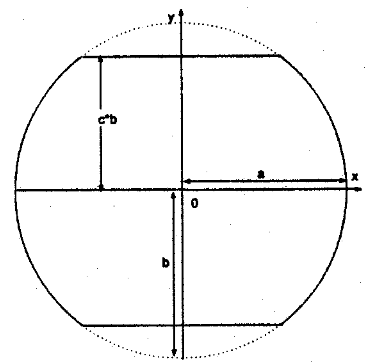
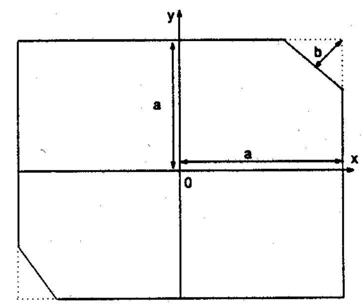


(a)



(b)

Fig. 1. Geometries and parameters of truncated waveguides. (a) Truncated circular or elliptical waveguide. (b) Truncated square waveguide.

II. Algorithm Development

Geometries of truncated circular (or elliptical) and square waveguides are shown in Fig. 1 where the conducting boundaries are depicted by solid lines. The cross section of a truncated circular (or elliptical) waveguide is a circle (or ellipse) with opposite parts removed in Fig. 1(a) where c is called the aspect ratio. In Fig. 1(b), the cross section of a truncated square waveguide is a square having diagonally opposite corners cut off and b represents the truncation parameter.

Details of the unified method for analyzing hollow conducting waveguides can be found in [10]. In order to compute cutoff wavenumbers in this paper, the Helmholtz equation together with the Dirichlet and Neumann boundary conditions for TM and TE modes needs to be solved, respectively. In the Cartesian coordinate system, the wave equation is

$$\nabla_T^2 \begin{bmatrix} E_z \\ H_z \end{bmatrix} + k_c^2 \begin{bmatrix} E_z \\ H_z \end{bmatrix} = 0 \tag{1}$$

where z denotes the longitudinal direction, the term x-y represents the transverse plane, ∇_T is the Laplacian operation in transverse plane, k_c stands for the cutoff wavenumber, and the longitudinal field $u(x, y)$ (E_z or H_z) is approximated by a series of polynomials Φ_i as follows

$$u(x, y) = \sum_{i=1}^{m} c_i \, \Phi_i \tag{2}$$

with m standing for the polynomial number. After substituting (2), the Rayleigh-Ritz procedure is used to rewrite the (1) into a generalized eigenvalue matrix equation. Once this generalized eigenvalue problem is solved, the cutoff wavenumbers and u can be obtained. For the TE case, we have

$$\Phi_i(x, y) = f_i(x, y) \tag{3}$$

and for the TM case

$$\Phi_i(x, y) = \psi(x, y) f_i(x, y). \tag{4}$$

The Rayleigh–Ritz procedure ensures the satisfaction of Neumman boundary condition for TE modes, and the function $\psi(x, y)$ is used to ensure the Dirichlet condition on the boundaries of the waveguide satisfied for TM modes explicitly.

The boundaries of truncated circular and truncated elliptical waveguide are described by

$$\psi(x, y) = \left[y^2 - (cb)^2\right] \left[\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 - 1\right]. \tag{5}$$

The truncated square waveguide is represented by

$$\psi(x, y) = \left[(x + y)^2 - 4(a - b \sin 45^\circ)^2\right] \times \left[\left(\tfrac{x}{a}\right)^{100} + \left(\tfrac{y}{a}\right)^{100} - 1\right]. \tag{6}$$

Applying (5) and (6) in the procedure of [10], the cutoff wavenumbers in above waveguides are obtained.

III. Numerical Results and Discussion

To confirm correctness and applicability of the method and code developed, we considered two examples, namely, a truncated circular waveguide and a truncated square waveguide. Cutoff wavenumbers in the two truncated waveguides are computed and compared with existing data available in literature.

First of all, cutoff wavenumbers of dominant modes in a truncated circular waveguide are computed using the present method. They are also compared with the results in [9], as shown in Table I. The number of polynomials used for TE modes is 30, except that it is 25 when $c = 0.1$. In the TM case, $20 \sim 30$ polynomials are used. Figs. 2 and 3 show cutoff wavenumbers of the higher TE and TM modes, respectively. Good agreements are observed for both TE and TM cases.

Secondly, the cutoff wavenumbers of TE modes in a truncated square waveguide are obtained using the unified method. The results obtained using the present method are compared in Fig. 4 with those of [4] obtained using the perturbation method. The agreement is excellent for small b. For larger b value, the deviation increases as expected, because the perturbation theory used in the literature becomes less accurate while our results are still accurate. The polynomial number used in this case is 25.

From the above two cases, it is seen in comparison that the unified method and program code we developed in this work are correct and applicable for computing cutoff wavenumbers in truncated waveguides.

An elliptical waveguide has the additional feature of eccentricity compared with a circular waveguide. There are two

TABLE I
Cutoff Wavenumbers of Dominant TE and TM Modes in Truncated Circular Waveguide

c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
k_c^{TE} (this)	1.5739	1.5814	1.5950	1.6146	1.6409	1.6744	1.7153	1.7623	1.8104	1.8412
k_c^{TE} ([9])	1.573	1.583	1.595	1.613	1.640	1.672	1.716	1.764	1.807	1.841
k_c^{TM} (this)	15.7865	8.0105	5.4681	4.2370	3.5285	3.0808	2.7859	2.5903	2.4699	2.4167
k_c^{TM} ([9])	15.73	8.010	5.470	4.235	3.528	3.081	2.785	2.588	2.462	2.405

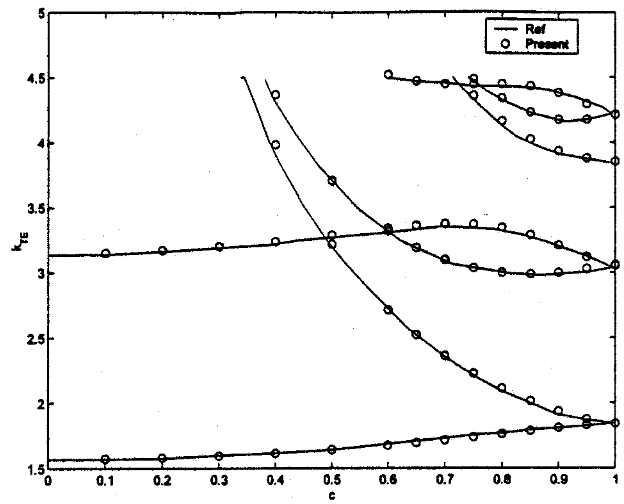


Fig. 2. Cutoff wavenumbers (of TE modes) in the truncated circular waveguide compared with results in [9].

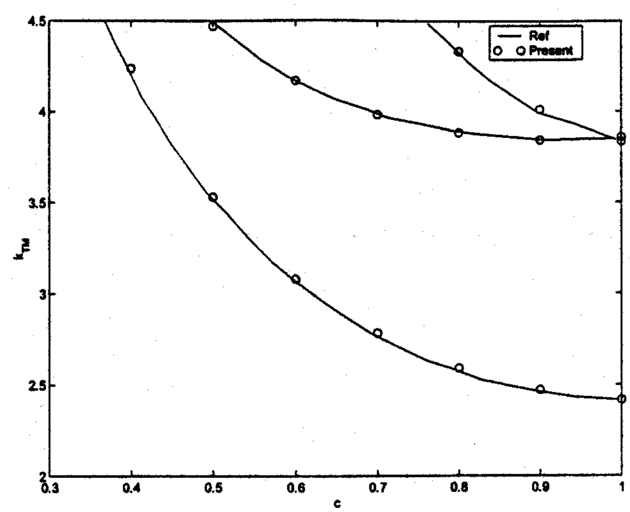


Fig. 3. Cutoff wavenumbers (of TM modes) in the truncated circular waveguide compared with results in [9].

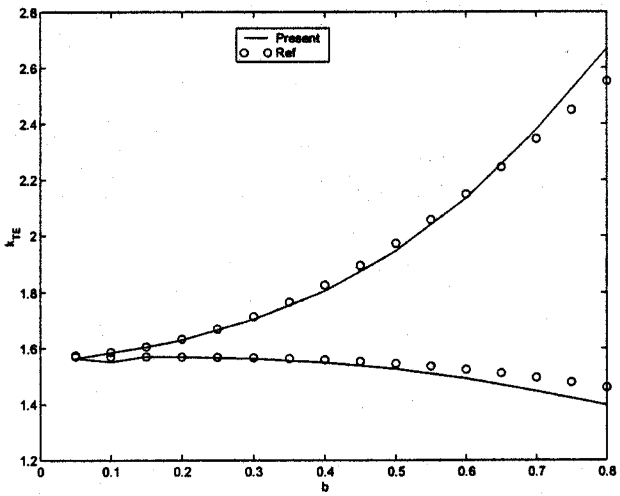


Fig. 4. Cutoff wavenumbers (of TE modes) in the truncated square waveguide compared with results in [4].

TABLE II
CUTOFF WAVENUMBERS OF DOMINANT TE AND TM MODES IN TRUNCATED ELLIPTICAL WAVEGUIDE ($e = 0.5$)

c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
k_{c1}^{TE}	1.5739	1.5814	1.5950	1.6147	1.6411	1.6748	1.7164	1.7651	1.8164	1.8512
k_{c2}^{TE}	1.8173	1.8260	1.8417	1.8644	1.8945	1.9328	1.9790	2.0310	1.9439	1.8512
k_{c1}^{TM}	15.8126	8.0619	5.5453	4.3354	3.6468	3.2187	2.9408	2.7604	2.6524	2.6065
k_{c2}^{TM}	18.2061	9.2049	6.2496	4.8060	3.9674	3.4331	3.0756	2.8343	2.6823	2.6056

types of truncated elliptical waveguide, namely, one with truncation located on the major axis and the other on minor axis. Subsequently, we employ the unified method to obtain cutoff wavenumbers in a truncated elliptical waveguide, as a new application. In Table II, cutoff wavenumbers of dominant TE and TM modes are given. The truncated ellipse is constructed from an ellipse with eccentricity (e) of 0.5. Two wavenumbers are considered, i.e., k_{c1} for the first case and k_{c2} for the second case. The polynomial number here is the same as that in the truncated circular waveguide.

When $c = 1$, the cross section becomes an ellipse. The analytic results of cutoff wavenumbers available in literature are 1.8510 and 2.5968 for TE and TM modes, respectively. The deviation between our results and analytic ones is less than 0.4%. When $c = 0.1$, the cross section corresponding to k_{c1} resembles a long rectangle. It looks like the truncated circular waveguide under the same condition. The cutoff wavenumbers for TE and TM modes are 1.5739 and 15.8126, very close to the corresponding results in Table I, respectively.

IV. CONCLUSION

In this paper, the cutoff wavenumbers of TE and TM modes in truncated circular and rectangular waveguides are obtained using the unified method alternatively. The truncated circular and square waveguides are reconsidered and their results available in the literature are compared with the data generated from the present code. It is found that they agree very well, which partially confirms the correctness and applicability of the present method and code. Then, a truncated elliptical waveguide, as a new structure which provides additional degrees of freedom in the design, is also analyzed using the present method.

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